MA 161 - Lesson 6 (2.4)

Today: * Infinite limits

* Vertical asymptotes

Office Hours: Monday, Wednesday, Friday 2:45pm - 4:15pm

Announcements: Exam 1 on Thursday 09/18, 8pm-9pm

HW5,6 Due: Tuesday 09/09

Quiz 3 (lesson 4): Tuesday 09/09

Warmup:

evaluate the following:

$$\frac{1}{0.1} = 10$$

$$\frac{1}{0.01} = 100$$

$$\frac{1}{0.001} = 1000$$

$$\frac{1}{0.0001} = 10000$$

$$\frac{1}{0.000001} = 1000000$$

$$\frac{1}{-0.1} = -10$$

$$\frac{1}{-0.01} = -100$$

$$\frac{1}{-0.001} = -1000$$

$$\frac{1}{-0.0001} = -10000$$

$$\frac{1}{-0.000001} = -1000000$$

Idea: $\frac{1}{\text{Small, +ve number}} = \text{Large \& +ve}$, $\frac{1}{\text{Small, -ve Number}} = \text{Large \& -ve}$

Infinite Limits

eg: $\lim_{x \to 7} \frac{1}{x-7}$ ⇝ goes to 0 as $x \to 7$

Right Limit: $\lim_{x \to 7^+} \frac{1}{x-7}$ as x approach 7 from Right

x	$x-7$	$\frac{1}{x-7}$
7.01	0.01	100
7.001	0.001	1000
7.00001	0.00001	100000

$\frac{1}{x-7}$ is getting Large & Larger & is positive

$\Downarrow$

Represent as $+\infty$, $\lim_{x \to 7^+} \frac{1}{x-7} = +\infty$

Left Limit $\lim_{x \to 7^-} \frac{1}{x-7}$ as x approaches 7 from Left

x	$x-7$	$\frac{1}{x-7}$
6.99	-0.01	-100
6.999	-0.001	-1000
6.99999	-0.00001	-100000

$\frac{1}{x-7}$ is getting Larger & Larger & -ve.

$\Downarrow$

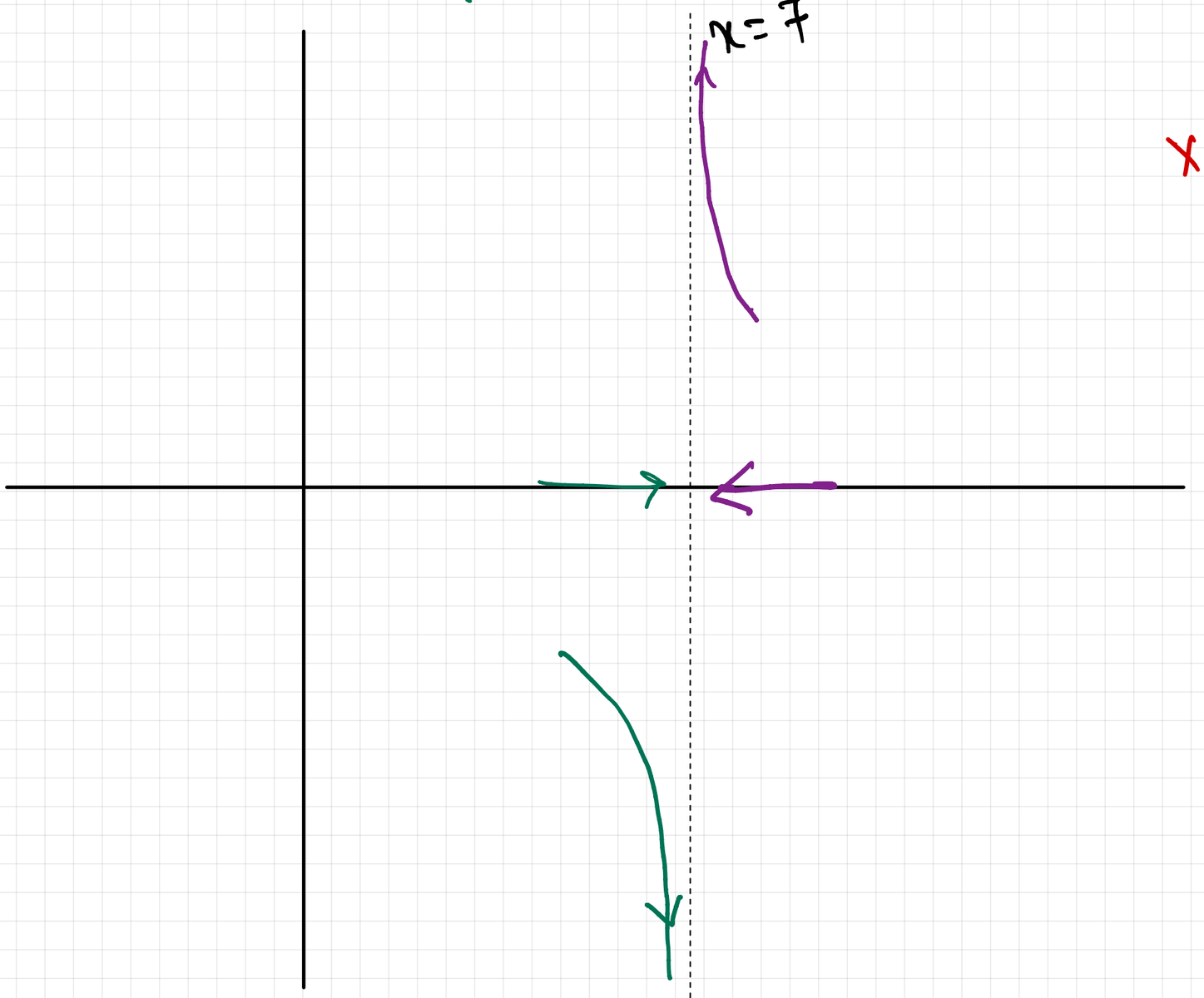
Represent as $-\infty$, $\lim_{x \to 7^-} \frac{1}{x-7} = -\infty$

Graph of $f(x) = \frac{1}{x-7}$, near $x = 7$

$\lim_{x \to 7^+} f(x) = +\infty$

$\lim_{x \to 7^-} f(x) = -\infty$

$\lim_{x \to 7} f(x) = \text{DNE}$



Line $x = 7$ is called Vertical Asymptote

for any function $f(x)$

$x=a$ is a vertical asymptote

if $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$

eg ①: $\lim_{x \to 7} \dfrac{x^2-4x-20}{x-7}$ → Plug in $x=7$ → $7^2-4(7)-20 = 1$

$x-7$ → approach 0 as x approach 7

$\lim_{x \to 7^+} \dfrac{x^2-4x-20}{x-7} \approx \dfrac{1}{\text{small, +ve}} = +\infty$

$\lim_{x \to 7^-} \dfrac{x^2-4x-20}{x-7} \approx \dfrac{1}{\text{small, -ve}} = -\infty$

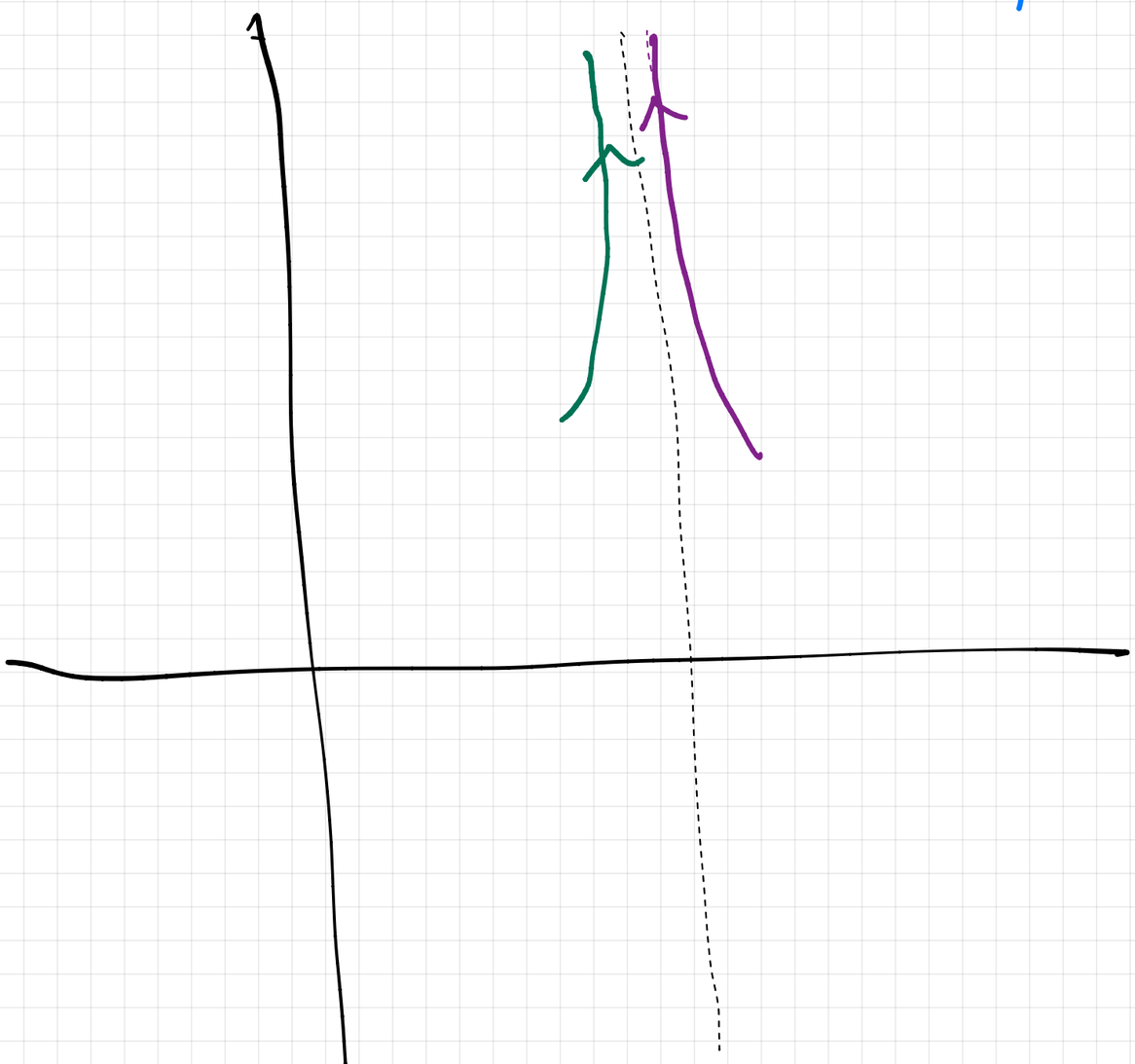
$\lim_{x \to 7} \dfrac{x^2-4x-20}{x-7}$ DNE

$x=7$ is a vertical Asymptote.

eg ② $\lim_{x\to 7} \frac{1}{(x-7)^2}$, Does $f(x) = \frac{1}{(x-7)^2}$ have a V.A?

$(x-7)^2 \rightsquigarrow$ small, +ve irrespective of $x \to 7^+$, or $x \to 7^-$

$\Rightarrow \frac{1}{(x-7)^2} =$ Large, +ve irrespective of $x \to 7^+$ or $x \to 7^-$



$\lim_{x\to 7^-} \frac{1}{(x-7)^2} = \lim_{x\to 7^+} \frac{1}{(x-7)^2} = +\infty$

$\lim_{x\to 7} \frac{1}{(x-7)^2} = +\infty$

$x=7$ is a V.A

eg ③ $\lim_{x\to 7^+} \dfrac{x+11}{\sqrt{x-7}}$

$\lim_{x\to 7^-} \dfrac{x+11}{\sqrt{x-7}}$

CANNOT plug in $x<7$, as domain $x>7$

$\lim_{x\to 7^-} \dfrac{x+11}{\sqrt{x-7}}$ DNE

$x\to 7^+$ ⇒ $x-7$ is small, +ve

$\sqrt{x-7}$ is small, +ve

$\lim_{x\to 7^+} \dfrac{x+11}{\sqrt{x-7}} \simeq \dfrac{18}{\text{small, +ve}}$ = Large, +ve $= +\infty$

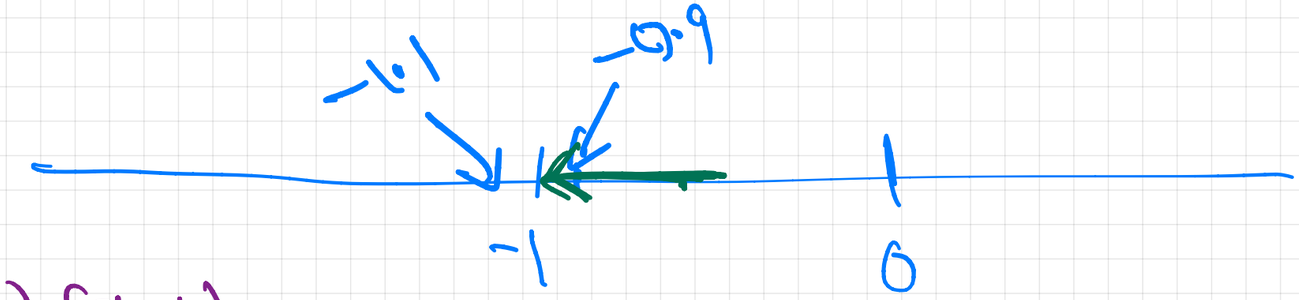
⇒ $x=7$ is a V.A.

eg④ Evaluate $\lim_{x \to -1} \frac{x^2-4x+3}{x^2-1}$

Factorize! $x^2-4x+3 = (x-3)(x-1)$

$x^2-1 = (x+1)(x-1)$

$\lim_{x \to -1^+} \frac{(x-3)(x-1)}{(x+1)(x-1)} \approx \frac{(-4)(-2)}{(\text{small, +ve})\ (-2)} = \text{Large, -ve} = -\infty$



$\lim_{x \to -1^-} \frac{(x-3)(x-1)}{(x+1)(x-1)} \approx \frac{(-4)(-2)}{(\text{small -ve})(-2)} = \text{Large, +ve} = +\infty$

$x = -1$ is a Vertical Asymptote

eg ⑤ Evaluate $\lim_{x\to 1} \frac{x^2-4x+3}{x^2-1}$

$$= \lim_{x\to 1} \frac{(x-3)(x-1)}{(x+1)(x-1)}$$

Can cancel $x-1$ in Num. & denominator when $x \neq 1$

$$= \lim_{x\to 1} \frac{x-3}{x+1} \overset{\text{plug in}}{=} \frac{-2}{2} = -1$$

$x=1$ is NOT a Verhical Asymptote

Be Careful, when both numerator & denominator $=0$.

eg(6): $f(x) = \dfrac{x^2-3x-28}{(x-a)(x-1)}$

for what values of a, $f(x)$ has a vertical asymptote at $x=a$

$x^2-3x-28 = (x-7)(x+4)$

$f(x) = \dfrac{(x-7)(x+4)}{(x-a)(x-1)}$

$a=7 \Rightarrow f(x) = \dfrac{(x-7)(x+4)}{(x-7)(x-1)}$

$\lim_{x\to 7} f(x) = \dfrac{11}{6}$

$x=7$ is Not V.A

$a=-4 \Rightarrow f(x) = \dfrac{(x-7)(x+4)}{(x+4)(x-1)}$

$\lim_{x\to -4} f(x) = \dfrac{11}{5}$

$x=-4$ is not VA

For Any $a \neq 7, -4$, You have V.A at $x=a$